Towards Meaningful Visual Abstraction of Mathematical Notation

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Abstract. The large variety of form factors to view web pages and the proliferation of the pinch-to-zoom paradigm requires web-content to adapt both font sizes and reflow to the requirements of diverse displays and varying magnification. For specialist web content, such as mathematical formulas, this is not straight forward. We present a first approach to *responsive equations*, a rendering method for MathML that adapts gracefully to small screens. The main idea is to reduce the size of equations by abstracting over well-defined parts of formulas without obscuring the overall structure of an expression. We achieve this by embedding a semantic structure into the MathML representation underlying the rendering process and by collapsing mathematically meaningful sub-expressions.

1 Introduction

Mathematical notation is a corner stone of scientific literature and with more and more teaching and research material being published in purely electronic form the electronic display of mathematics has become an important topic. MathML is the only specialist markup language that has been made part of the HTML5 and epub3 standard. While this was an important first step, there is still a long way to go before Mathematical notation becomes a first-class citizen of the web. Not only is MathML not implemented in all browsers or eBook readers, forcing developers and content providers to use polyfill solutions like MathJax [4] to ensure that content renders consistently across all platforms, but also the production of mathematical content is still mainly done in a “print-first” manner, giving little thought to the special requirements and opportunities of electronic display. For example, rigid, tabular layout is ported directly from print, making it difficult to adjust its display for different form factors or to exploit features like zooming or panning.

In the past five years, the notion of *responsive web design* (RWD), introduced in [1] (cf. also [13]), has been firmly established as the dominant design paradigm on the web. RWD aims to dynamically optimise the layout of a page depending on the capabilities of the end user’s device. The dominating features of RWD are fluid grids for content, flexible images, and media queries. The related (and older) concept, “progressive enhancements”, cf. [8], leverages feature and (client or server-side) user-agent detection to enrich the content and rendering.

While RWD offers a firmly established methodology, tools, and workflows for content such as text, menus, and widgets, the notion of responsive design for complex content fragments is still in its infancy. This includes responsive tables [14] and responsive SVG [10]. In addition, data visualisation tools such as d3js allow dynamic rendering of scientific data that shares some of the traits of responsive design as their dynamic nature enables developers to modify them on the fly, cf. [9].

For MathML, native browser implementations, i.e., Gecko and WebKit, do not yet provide line-breaking support. Under these circumstances, it might not be surprising that the notion of responsive rendering has not even been attempted yet; that is, no new visual concepts have been explored that work with the fabric of the web, similar to the attempts for tables and SVG. Unlike regular text that can be easily broken down into paragraphs, sentences, etc., regardless of their meaning, it is difficult to identify meaningful components in mathematical formulas without having some information on their

⋆ This work was partially supported by the Alfred P. Sloan Foundation.

actual semantics. In particular, issues like line breaking, which are straight-forward for regular text, are difficult for MathML renderers, as large expressions often have to be broken over multiple lines; this makes the formula hard to read, and when done badly, can render it practically meaningless.

Consider the example in Table 1 that was extracted from an answer on math.stackexchange.com, [5]. Not only is the expression rather large and unwieldy, it also is given in a pre-formatted tabular style that makes it difficult to decide on both visually pleasing and mathematically accurate line breaks. Moreover, some of the sub-expressions, like the presentation annotation with underbrace or the wide fration elements, do not lend themselves at all to line breaking. Consequently display and reading of the expression on a small form factor can be very awkward.

$$
\begin{aligned}
I_\nu(\nu^{-1},1) &= \underbrace{\frac{\pi^2}{4}\ln\left(\frac{(1+\nu)^{1+\nu}}{\nu^\nu}\right)-\frac{7\zeta(3)}{8}\nu}_{\text{Let this be C}} + 2\int_1^{\frac{1-\nu}{1+\nu}}\frac{\chi_3(v)}{(1+v)^2}\mathrm{d}v \\
&= C - \left.\frac{2\chi_3(v)}{1+v}\right|_1^{\frac{1-\nu}{1+\nu}} + 2\int_1^{\frac{1-\nu}{1+\nu}}\frac{\chi_2(v)}{v(1+v)}\mathrm{d}v \\
&= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8} - \left.2\chi_2(v)\ln(1+v)\right|_1^{\frac{1-\nu}{1+\nu}} + \int_1^{\frac{1-\nu}{1+\nu}}\frac{\ln(1+v)\ln\left(\frac{1+v}{1-v}\right)}{v}\mathrm{d}v \\
&= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8} + 2\chi_2\left(\frac{1-\nu}{1+\nu}\right)\ln\left(\frac{1+\nu}{2}\right) + \frac{\pi^2}{4}\ln 2 \\
&\quad + \frac{1}{2}\int_1^{\frac{1-\nu}{1+\nu}}\frac{\ln^2(1+v)-\ln^2(1-v)+\ln^2\left(\frac{1-v}{1+v}\right)}{v}\mathrm{d}v
\end{aligned}
$$

Table 1. Example of mathematics "in the wild" taken from math.stackexchange.com.

We currently address these issues within MathJax. In particular, we are developing new rendering methods for mathematical notation that allow for a responsive reading experience regardless of platform or form factor. The basic idea is to enrich standard MathML notation with semantic information computed by a heuristic interpretation of the Presentation MathML element. We thereby use a coarse-grained semantic model that aims to capture the maturity of mathematical notation, including interpretations for purely presentation artefacts, such as the underbrace in the example above, rather then determining a precise mathematical semantic that would severely restrict the types of expressions we could handle. The semantics effectively provide the original MathML expression with a secondary structure; and while the expression can be rendered as usual, the semantic markup can be exploited for a number of enhanced rendering effects. As a simple application the semantic information can be exploited for better line-breaking. As an advanced application we enable truly responsive notation that will collapse equations to fit the screen size by using semantic knowledge to abstract over parts of the equation. The responsive rendering provides a good overview of an expression and a simple UI allows readers to fully explore its content when needed. This design not only breaks with traditional layout paradigms to adapt to reading habits on small screens, but also has the potential to construct summaries of expressions on multiple layers, which can be exploited by assistive technology tools such as screen readers.

2 Semantic Enrichment

The semantic enrichment procedure is based on a semantic tree transformation for MathML elements that was originally designed and implemented in the context of making Mathematical notation accessible in the screenreader ChromeVox [11]. It constructs a semantic interpretation of an expression purely by analysing the syntactic structure of the MathML elements, while aiming to stay faithful to the given notation, not fixing too much of the semantics, which could lead to false interpretations. It consequently provides a much more shallow interpretation than a full blown semantic markup language like Content MathML [2]. In this sense it is more similar to LaTeXML's internal format [7] or SnuggleTeX's initial semantic enrichment process [6], which concentrate primarily on the interpretation of symbols occurring in expressions. It therefore furthers our goal to enhance the presentation markup, rather than replacing it by a purely semantic representation. Nevertheless we believe that our current set of heuristics can be viewed as an intermediate step towards full semantic markup and, provided with additional context or domain information, could lead to a translation into Content MathML or a similar format.

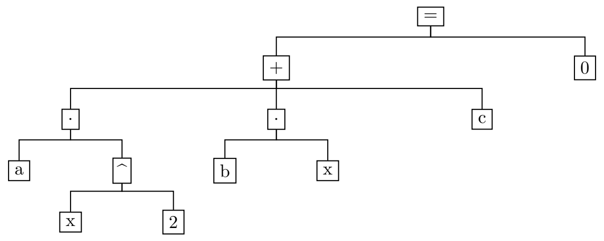
We will first briefly sketch the major ideas of the semantic tree and then describe how its information is integrated into existing MathML structures.

2.1 Semantic Tree

The main problem for semantic enrichment of Presentation MathML is to transform its flat structure into one that correctly determines the scope of operators, relations, etc. Our approach aims to represent a formula in a semantic tree structure akin to a term tree. The semantic tree is assembled bottom-up, where we first classify the single components of an expression, giving each an immutable type and a mutable role. The former aims to capture the basic nature of the symbol, while the latter is used to describe the role of a symbol in the context of the formula. For example, f, which has the type of identifier gets a default role of Latin letter assigned, while no additional information is known. Once more knowledge on its semantic meaning is available, its role is refined. For example, in the expression $f(x)$ it would get the role of a function, while its role remains unchanged in $f + g$.

A central heuristic then builds term trees from flat structures by promoting relations and defining operator precedence orders as well as determining properly delimited structures. As an example of this heuristic we observe how the quadratic equation $ax^2 + bx + c = 0$ is rewritten from its Presentation MathML representation into its semantic interpretation below:

```
<math>
  <mi>a</mi>
  <msup>
    <mi>x</mi>
    <mn>2</mn>
  </msup>
  <mo>+</mo>
  <mi>b</mi>
  <mi>x</mi>
  <mo>+</mo>
  <mi>c</mi>
  <mo>=</mo>
  <mn>0</mn>
</math>
```



Observe that the transformation tries hard to recognise elided multiplications. In addition, our procedure contains a number of heuristics, in particular to 1. determine potential function applications, 2. break up symbol sequences into elided products, 3. recognise scope and nesting of big operators (e.g., sums, integrals), 4. distinguish tables into matrices, vectors, and case statements, 5. combine punctuated expressions and determine the meaning of ellipses.

Technically the tree is constructed by analysing MathML elements, interpreting their type, role and font, and turning them into semantic nodes, with parent pointer and a variable number of children. In addition we have a notion of content elements for each node. This is a possibly empty list of semantic nodes that are combined or abstracted over by this particular node.

For example, a node representing the application of a single operator like +, to a variable number of summands, will have only the semantic nodes representing the summands as children, while retaining all the intermediate occurrences of + in its list of content nodes. This allows us to keep a connection between nodes of the semantic tree and elements in the original MathML structure for tasks like synchronised highlighting or the semantic enrichment we will discuss in the next section.

2.2 Embedding into MathML

The basic idea of embedding the semantic tree into MathML is by modelling the components of the tree via additional data attributes in the single elements of the MathML expression. We have decided on data attributes over alternative means, e.g., exploiting Presentation MathML's semantics tag, RDFa, or micro data, for a number of reasons: primarily, we want to embed the semantic structure into the presentation element to provide a different view on the MathML expression, rather than having a new structure in parallel instead. Thereby data attributes provide a fast means of retrieving information from the DOM, which is fully consistent with HTML5 practices.

In practice, we add new data attributes to reflect both content and structure of the semantic tree. The former are attributes reflecting type, role and font information stored in each node of the tree. The latter effectively provide each node with a unique semantic id, and, if necessary, a parent pointer and lists of pointers to children and content nodes. In addition we have attributes that provide administrative information with respect to artefacts that have been introduced or omitted due to the mapping onto the MathML expression.

In the majority of cases the embedding is straight forward, with as little modification to the original MathML expression as possible. However, this can not always be maintained for more complex structures. As a consequence we have the following cases to consider:

Extra Groupings are often necessary to break up flat rows of operators and identifiers in order to reflect the term tree structure of the inferred semantics. These are achieved by grouping elements inside additional `mrow` elements to reflect the layers of the term tree.

Added Invisible Elements are necessary when the semantic interpretation determines elided operators, such as implicit multiplications or function applications. Additional `mo` elements will be introduced containing unicode characters like invisible times, invisible comma or function application.

If we consider again the example of the quadratic equation, the enriched Presentation MathML will look like this:

```
<math type="relseq" role="equality" id="16" children="15,10" content="9">
 <mrow type="infixop" role="addition" id="15" children="12,14,8" content="4,7" parent="16">
  <mrow type="infixop" role="implicit" id="12" children="0,3" content="11" parent="15">
   <mi type="identifier" role="latinletter" id="0" parent="12">a</mi>
   <mo type="operator" role="multiplication" id="11" parent="12" added="true">&#x2062;</mo>
   <msup type="superscript" role="latinletter" id="3" children="1,2" parent="12">
    <mi type="identifier" role="latinletter" id="1" parent="3">x</mi>
    <mn type="number" role="integer" id="2" parent="3">2</mn>
   </msup>
  </mrow>
  <mo type="operator" role="addition" id="4" parent="15">+</mo>
  <mrow type="infixop" role="implicit" id="14" children="5,6" content="13" parent="15">
   <mi type="identifier" role="latinletter" id="5" parent="14">b</mi>
   <mo type="operator" role="multiplication" id="13" parent="14" added="true">&#x2062;</mo>
   <mi type="identifier" role="latinletter" id="6" parent="14">x</mi>
  </mrow>
  <mo type="operator" role="addition" id="7" parent="15">+</mo>
  <mi type="identifier" role="latinletter" id="8" parent="15">c</mi>
 </mrow>
 <mo type="relation" role="equality" id="9" parent="16">=</mo>
 <mn type="number" role="integer" id="10" parent="16">0</mn>
</math>
```

Observe that the element now contains both extra groupings and invisible times applications, which are marked as newly added.

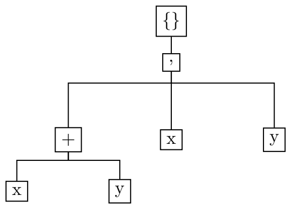
Empty Elements need to be added in case the semantic interpretation contains mandatory, possibly empty elements, that might not be present in the presentation. As an example, consider an integral expression consisting of three parts: (1) the integration sign, possibly with embellishments like limits, (2) the integrand, and (3) the integral variable. If either of the latter two components is not present, they are represented by an empty element. This is reflected by introducing an empty `mrow` element in the Presentation MathML tree, which does not change the visual appearance of the rendered expression.

Collapsed Elements can occur when the semantic interpretation introduces additional structure that can not easily be reflected in the presentation element without potentially altering the visual rendering. For example, when semantically indicated, combined sub- and superscript elements will be represented as subscript with a superscript. This leads to an additional layer in the tree that is not present in the presentation element, and that can not be introduced without changing rendering behaviour. Consequently, the omitted structure is given in a Lisp-like notation to avoid unconnected pointers and to ease potential reconstruction of the semantic from the attributes alone.

Special Cases are complex elements such as `mfenced` or `mmultiscripts`. For example, in the case of the former the semantic interpretation needs to consider components of the expression that are only given implicitly via element attributes. In detail, `mfenced` allows an author to specify both fences as attributes `open` and `close`; absence of fences is enforced by giving empty string arguments as attribute values, while omitting the attributes leads to parentheses being inserted as default. Similarly, a `separators` attribute allows to specify a list of characters that should be rendered between children of the `mfenced` element. For semantic interpretation these attributes need to be expanded and represented explicitly in the semantic tree, which in many cases makes it difficult and sometimes impossible to map the semantic structure back onto the original MathML element. Consequently, the original `mfenced` element is replaced by an explicit `mrow`, with opening and closing fences as well as separators modelled as proper new MathML elements. While this might lead to a considerably altered MathML expression, it is necessary to capture fully semantic meaning, as we observe with the following example, where the `separators` attribute is abused to model operations as well:

$$\{x + y, x, y\}$$

```
<mfenced
  open="{" close="}"
  separators="+,">
   <mi>x</mi>
   <mi>y</mi>
   <mi>x</mi>
   <mi>y</mi>
</mfenced>
```



```
<mrow type="fenced" role="leftright" id="11" children="10" content="7,8">
 <mo type="fence" role="open" id="7" parent="11" added="true">{</mo>
 <mrow type="punctuated" role="sequence" id="10" children="9,5,2,6,3" content="5,6" parent="11">
  <mrow type="infixop" role="addition" id="9" children="0,1" content="4" parent="10">
   <mi type="identifier" role="latinletter" id="0" parent="9">x</mi>
   <mo type="operator" role="addition" id="4" parent="9" added="true">+</mo>
   <mi type="identifier" role="latinletter" id="1" parent="9">y</mi>
  </mrow>
  <mo type="punctuation" role="comma" id="5" parent="10" added="true">,</mo>
  <mi type="identifier" role="latinletter" id="2" parent="10">x</mi>
  <mo type="punctuation" role="comma" id="6" parent="10" added="true">,</mo>
  <mi type="identifier" role="latinletter" id="3" parent="10">y</mi>
 </mrow>
 <mo type="fence" role="close" id="8" parent="11" added="true">}</mo>
</mrow>
```

3 Responsive Equations

Responsive design enhances a core feature of HTML: reflow. Originally focusing on re-arranging and optimising content, new tools transform the content itself, e.g., cropping images[12], abstracting icons [10], or modifying tables [14].

Reflowing mathematics poses a great challenge as it combines the properties of text, tables, and graphics into a singular problem. While good line-breaking algorithms exist for print, they are often counter-productive on the web, damaging legibility of larger equations beyond repair. The problem is exacerbated by the fact that content is created with print in mind, manually fitting it to page dimensions – manual line breaks, arrangements across tabular layout, and other *tweaks* prevent a sensible reflow.

We leverage the semantic enrichment to create *responsive equations*, a completely novel way of dynamically presenting math on small screens. Before approaching rendering on small screens, a crucial consideration lies in the user story for accessing documents with mathematical content on small devices in general. In other words, the design has to adapt to the specific use case that an author and designer has identified.

Our approach is targeted at what might be called *casual* reading. This includes scenarios such as browsing through lists of recent publications (e.g., repository or journal news feed), cross-reading a publication, and looking up references. In these use cases, the reader's primary interest does not lie in being able to fully access every MathML fragment immediately. Rather, in this scenario we need to reduce visual noise to enable users to efficiently reach their goal. At the same time, the mathematical fragment cannot simply be hidden as it often serves a structural role in the overall content and can also be the specific target of the user (e.g., a labelled equation). This might be compared to the rendering of maps, though in reverse: when a user zooms out, they do not want the map to be cluttered with pointers to the most detailed level of information, and yet they need to be able to access that level of detail should they need to.

Therefore, our current approach is (a) to collapse and re-arrange sub-expressions on small screens to provide the reader with a meaningful overview of the expression and (b) to implement an interface for exploration of collapsed equations. We first discuss the machinery for the exploration and present examples in the next section.

3.1 Exploring Equations

Since an equation in a collapsed state hides parts of its content, the responsive rendering mode requires a user interface for exploration.

For the first iteration of this project we have chosen a straight-forward implementation using MathML's `maction` element with `actiontype` set to `toggle`, cf. [2, 3.7.1]. This will allow users to explore the content using click, keyboard, and touch events. The `maction` elements are nested so that only the next level of the collapse is revealed.

The element indicating collapsed content is currently a simple Unicode construction, ◀X▶, with X indicating the top-level structure that was collapsed. Currently, we differentiate the following structures:

- Matched fences: ◀[)▶ etc.
- Function application: ◀f()▶
- Fraction: ◀/▶
- Surds: ◀√▶
- Scripts: ◀□˙▶, ◀□.▶, ◀□:▶
- Large operators: ◀Σ▶ etc.
- Repeated operators: ◀+▶ etc.
- Long identifier: ◀x▶
- Long number: ◀#▶
- Long text: ◀. . .▶
- Vector: ◀⟨:⟩▶
- Square Matrix: ◀[::]▶
- Row vector: ◀⟨· · ·⟩▶
- Column vector: ◀⟨⁝⟩▶
- Unknown matrix: ◀(::)▶
- Punctuated text: ◀...▶
- Punctuated gen.: ◀,▶ etc.

Although currently fixed, authors will be able to customise these placeholders in the future, using Unicode, CSS, or SVG.

While `maction toggles` are simple and standard, the user experience is not ideal. For example, it is too easy to accidentally trigger a collapse of parts of the equation. Similarly, it is difficult to quickly un-collapse the entire equations. As we gain experience, we hope to be able to augment the regular toggle and feed that experience back into the development of the MathML specification.

3.2 Complexity Measure

Since collapsing is intended to shorten an expression when space is at a premium, one approach to determining what to collapse would be based on the widths of the various terms. For our algorithm, however, the widths of the terms are not yet known (as the widths are not computed until the terms are typeset for output, while the potential collapses are determined before the output process begins). Instead of width, our approach is to use a measure of complexity of the terms as a surrogate for width. (As a side-effect, this makes it possible to collapse for reasons other than saving space – see the section on size versus content in section 5 below.)

Each term in the MathML expression is assigned a complexity value based on the length of its content or the complexity of its children. A MathML token element (like an identifier or a number) is given a complexity determined by the number of characters in the text of the element; a longer identifier or number has a higher complexity (since we want complexity to act as a replacement for width). Other elements, like square roots, or fractions, have their complexity determined by combining the complexity of their child nodes.

For example, an expression that consists of a combination of three terms will have a complexity that is the sum of the complexities of the individual terms and the complexities of the operators that joins them. Because we want combinations that involve more terms to be more likely to collapse than those with few terms, the complexity is augmented by a factor based on the number of children (regardless of the complexities of the individual terms). So in the sum $a + b$, suppose the a, $+$, and b are each assigned a complexity of 1, then the complexity of the sum might be 6 (3 for the sum of the complexities of the element of the sum, and 3 more for the fact that there are three children of the expression).

In a similar fashion, a fraction might have complexity that is the sum of the complexities of the numerator and denominator, plus an additional amount for being a fraction, while a square root's complexity might be the complexity of its argument plus something more for being a square root.

In this way, each term in the MathML tree is given a complexity. This value is used to decide whether the term should be collapsible or not. The cut-off value that determines this is based on the semantic type of the MathML element. For example, if the cut-off value for a sum was set to 12, then the sum $a + b + c$ (with a complexity of 10) would not be collapsible, while $a + b + c + d$ (with a complexity of 14) would be, and $100 + 100 + 100$ (with a complexity of 14.5) would also be collapsible.

All the parameters involved in the complexity computations (e.g., the weight of each character in a token element, the weights of child terms, the extra amount for a fraction or root, and the cut-off values for collapsing) are stored in tables that can be adjusted by the page author, should the default settings not provide suitable collapses for the equations in use on the page.

4 Examples and Initial Results

For development and experimentation we avoid manual doctoring of examples, but look for examples "in the wild"; this aligns with our focus on handling arbitrary content well, not on handling well-prepared content excellently. We therefore give an overview of the core features and challenges of our approach, by further exploring the example from the introduction that we have found on math.stackexchange.com as the following original LaTeX code:

```
\begin{align}
I_\nu(\nu^{-1},1)
&=\underbrace{\frac{\pi^2}{4}\ln\left(\frac{(1+\nu)^{1+\nu}}{\nu^\nu}
\right)-\frac{7\zeta(3)}{8}\nu}_{\text{Let this be
C}}+2\int^\frac{1-\nu}{1+\nu}_1\frac{\chi_3(v)}{(1+v)^2}{\rm d}v\\
&=C-\left.\frac{2\chi_3(v)}{1+v}\right|^\frac{1-\nu}{1+\nu}_1+2\int^\frac{1-\nu}
{1+\nu}_1\frac{\chi_2(v)}{v(1+v)}{\rm d}v\\
&=C+(1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right)-\frac{7\zeta(3)}{8}
-\left.\color{white}{\frac{1}{1}}2\chi_2(v)\ln(1+v)\right|^\frac{1-\nu}{1+\nu}
_1+\int^\frac{1-\nu}{1+\nu}_1\frac{\ln(1+v)\ln\left(\frac{1+v}{1-v}\right)}{v}{
\rm d}v\\
&=C+(1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right)-\frac{7\zeta(3)}{8}
+2\chi_2\left(\frac{1-\nu}{1+\nu}\right)\ln\left(\frac{1+\nu}{2}\right)+\frac{
\pi^2}{4}\ln{2}\\
&\ \ \ \
+\frac{1}{2}\int^\frac{1-\nu}{1+\nu}_1\frac{
\ln^2(1+v)-\ln^2(1-v)+\ln^2\left(\frac{1-v}{1+v}\right)}{v}{\rm d}v
\end{align}
```

Note that the source contains a few artefacts such as an *invisible* (in fact, white) fraction as well as manual spacing.

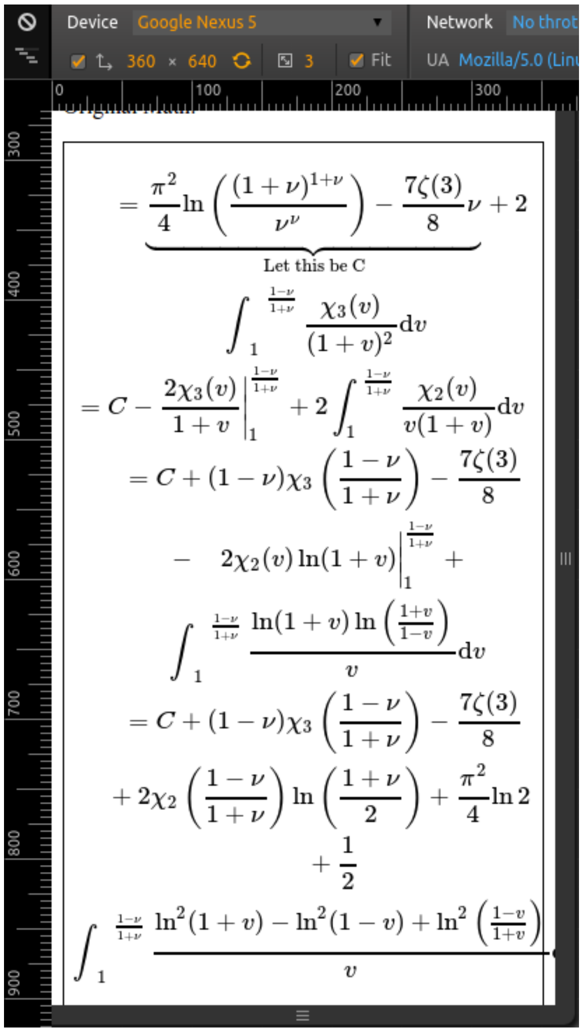
To provide a real-world experience, we have simulated a Nexus 5 device using the developer tools of the Chrome browsers, with a simulated display size of 360px × 640px. In our sample HTML page, the font size for the mathematical fragment is ∼18.5px.[1]

4.1 Line-breaking

Applying line-breaking programmatically to this kind of equation is challenging. Align environments have to be interpreted as `mtable` elements as they might have equation labels attached to them which can only be implemented inside `mtable` structures. Line-breaking does not touch the table structure and can only perform line-breaks within tables. The results are often visually complicated.

In Figure 1 we see the result of rendering with line-breaking[2] (no enrichment applied) on an simulated smart-phone device. The first observation is that the initial column takes up most of the screen already, pushing the second column outside the viewport. In other words, line-breaking fails fundamentally to fit the content on the screen, just as with other table structures.

For the screenshot we simulated a swipe to bring the second column into view. Here we encounter additional problems. Since line-breaking occurs within each cell, there is no way to align the breaks across rows. Instead, iterated line-breaks in different cells produce an uneven rendering. In addition, we see how the quality of the markup negatively affects the layout, e.g., in the top row the linebreak splits a product that forms a single summand. In short, the result is very difficult to process for the reader. See also subsection 4.3 for further discussion of line-breaking.



Device Google Nexus 5 · Network No throt

360 × 640 · 3 · Fit · UA Mozilla/5.0 (Lin

$$= \underbrace{\frac{\pi^2}{4}\ln\left(\frac{(1+\nu)^{1+\nu}}{\nu^\nu}\right) - \frac{7\zeta(3)}{8}\nu + 2}_{\text{Let this be C}}$$

$$\int_1^{\frac{1-\nu}{1+\nu}} \frac{\chi_3(v)}{(1+v)^2}\mathrm{d}v$$

$$= C - \frac{2\chi_3(v)}{1+v}\bigg|_1^{\frac{1-\nu}{1+\nu}} + 2\int_1^{\frac{1-\nu}{1+\nu}}\frac{\chi_2(v)}{v(1+v)}\mathrm{d}v$$

$$= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8}$$

$$- \quad 2\chi_2(v)\ln(1+v)\bigg|_1^{\frac{1-\nu}{1+\nu}} +$$

$$\int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln(1+v)\ln\left(\frac{1+v}{1-v}\right)}{v}\mathrm{d}v$$

$$= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8}$$

$$+ 2\chi_2\left(\frac{1-\nu}{1+\nu}\right)\ln\left(\frac{1+\nu}{2}\right) + \frac{\pi^2}{4}\ln 2$$

$$+ \frac{1}{2}$$

$$\int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln^2(1+v) - \ln^2(1-v) + \ln^2\left(\frac{1-v}{1+v}\right)}{v}$$

Fig. 1. Screenshot: linebreaking

4.2 Collapsing

In comparison, the (maximally collapsed) responsive rendering of the equation will look like the following.

◀f()▶ = ◀+▶
= ◀+▶
= ◀+▶
= ◀+▶ + ◀·▶ + ◀·▶
◀…▶

[1] The precise font size is determined on the fly to match ex-heights with the surrounding font, which may differ across devices.

[2] MathJax implements most of the MathML specification regarding line-breaking.

With a width of ~258px, this rendering fits well even on very small screens and avoids additional noise. While it collapses the details, it does so gracefully, retaining the fundamental layout.

Since the screen size of the simulated Nexus 5 device is actually larger, our current implementation will automatically expand sub-expressions to expose as much detail as possible as seen in Figure 2.

As described earlier, our current implementation uses standard `maction` elements to enable the user to explore the equation. Each placeholder serves as a toggle, expanding nested layers of the equation. To provide a final example, a reader might then want to explore particular aspects of the expression, such as the evolution of the integral term; a sample exploration is captured in Figure 3.

As can be seen, the user interface for exploration is still in its infancy and we do not, in fact, expect to come up with a smooth implementation. From personal experience, this rendering changes the 'reading of' (or interaction with) an expression significantly and only real user feedback will determine which aspects need to be modified.

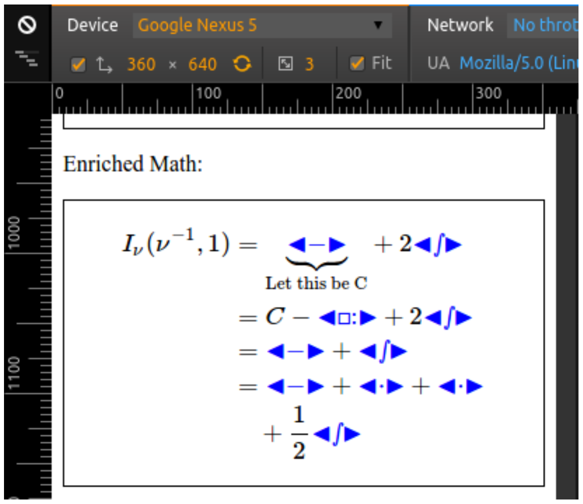


Fig. 2. Screenshot: responsive rendering

4.3 Improving default rendering

A significant problem in professional publishing workflows lies in the low quality of Presentation MathML markup. MathML fragments are often produced by third-party vendors who convert documents into XML. MathML fragments are either generated from other input formats such as TeX/LaTeX code or manually recreated from other renderings (image, pdf). In the latter case, it is unrealistic to expect such conversion specialists to have enough domain specific knowledge to capture nuances in the semantics and presentation of the mathematical fragments. Even in the case of a converter being used, the source markup is often of low quality, e.g., TeX lacking appropriate `\right` `\left`). This leads to very *flat* MathML fragments. Ultimately, the results of this process are too often of poor quality.

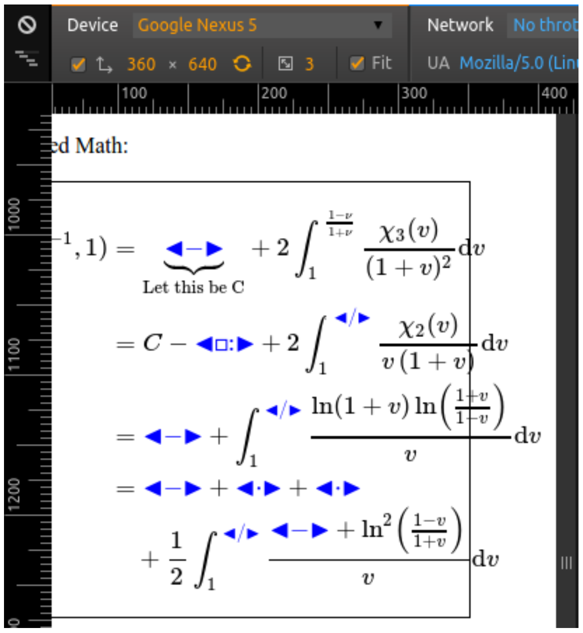


Fig. 3. Screenshot: exploring responsive rendering

The enriched Presentation MathML can resolve some of these issues and thus improve rendering in general. This is due to gently enhancing the MathML structure, e.g., with missing mrows, matched fences, and so on.

The following example reconstructs the first few terms of [3, Eq. 17].

$$\Bigg|\tau_0\Bigg| = \Bigg|\sum_m \Bigg(a_b + b_m\Bigg)\Bigg|$$

The reason for these oversized fences lies in the *flat* structure of the underlying Presentation MathML. MathML specifies that stretchy fences should match the height of the tallest sub-expression within an mrow. The lack of any grouping in the MathML source forces renderers to have all fences match the size of the $\sum$.

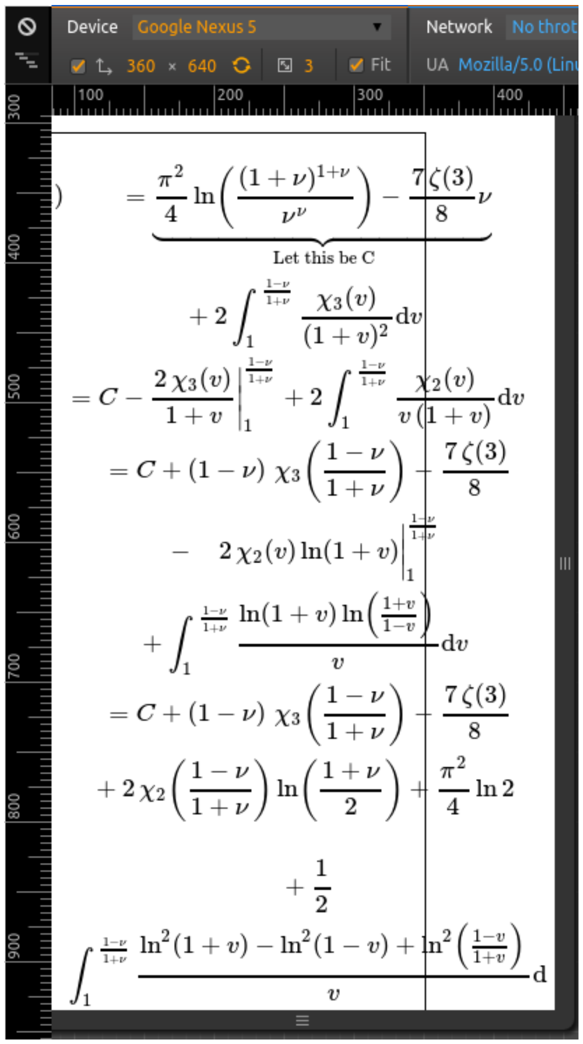
After enriching the MathML, matching fences are identified and additional mrows introduced, leading to improved rendering equivalent to the following.

$$|\tau_0| = \left|\sum_m (a_b + b_m)\right|$$

As this may seem like a mild effect we can only stress how wide-spread and persistent such problems are in published MathML.

Additionally, these subtle improvements to the markup make it easier for line-breaking algorithms to determine good breakpoints. To revisit our example, Figure 4 consists of a screenshot with line-breaking after semantic enrichment. Note how the second line of the first cell and the third line of the third cell have improved by grouping the multiplication as well as addition correctly.

Anecdotal evidence from sharing our work with other researchers suggests this rendering could have the unexpected side effect to alert authors to bad practices in their markup. That is, researchers responded to the new rendering by stating that they want to author in a way that renders well responsively. While clearly biased, it will be an interesting avenue for further testing.


Device Google Nexus 5
Network No throt
360 × 640 3 Fit
UA Mozilla/5.0 (Lin

$$) = \underbrace{\frac{\pi^2}{4}\ln\left(\frac{(1+\nu)^{1+\nu}}{\nu^\nu}\right) - \frac{7\zeta(3)}{8}\nu}_{\text{Let this be C}}$$
$$+2\int_1^{\frac{1-\nu}{1+\nu}} \frac{\chi_3(v)}{(1+v)^2}dv$$
$$= C - \frac{2\chi_3(v)}{1+v}\Big|_1^{\frac{1-\nu}{1+\nu}} + 2\int_1^{\frac{1-\nu}{1+\nu}} \frac{\chi_2(v)}{v(1+v)}dv$$
$$= C + (1-\nu)\,\chi_3\left(\frac{1-\nu}{1+\nu}\right) + \frac{7\zeta(3)}{8}$$
$$- \quad 2\chi_2(v)\ln(1+v)\Big|_1^{\frac{1-\nu}{1+\nu}}$$
$$+\int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln(1+v)\ln\left(\frac{1+v}{1-v}\right)}{v}dv$$
$$= C + (1-\nu)\,\chi_3\left(\frac{1-\nu}{1+\nu}\right) + \frac{7\zeta(3)}{8}$$
$$+2\chi_2\left(\frac{1-\nu}{1+\nu}\right)\ln\left(\frac{1+\nu}{2}\right) + \frac{\pi^2}{4}\ln 2$$
$$+\frac{1}{2}$$
$$\int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln^2(1+v) - \ln^2(1-v) + \ln^2\left(\frac{1-v}{1+v}\right)}{v}d$$

Fig. 4. Screenshot: linebreaking without enrichment

5 Discussion and Future Work

Although the main features of our approach are implemented and demonstrate the power of a responsive mode for mathematical expressions, there are still a number of issues that need to be solved, for both the semantic enrichment and the responsive equations. For the former we need to improve in particular heuristics for matching brackets and dealing with embellished fences. For the latter, we need to deal more elegantly with multiline equations and we want to experiment with ways to indicate where formulas can be collapsed to prevent users from randomly clicking on symbols.

Some other questions we will be looking at in the future are:

Simultaneous Expansion Currently equations are collapsed and expanded step-wise, independently in different components of a formula. But actions could be coordinated, that is, the expansion or collapse of a particular sub-formula would trigger the corresponding action on similar sub-formulas elsewhere in the equation. Coordinating actions would be particularly important when working on small form factors to support meaningful zooming and panning.

Measures of Complexity The measure of complexity we use is based mainly on an attempt to determine visual layout size. In the future we want to experiment with different measures that can capture other notions, such as giving precedence to certain operators or expressions, thereby defining a measure of interestingness of a sub-formula. Consequently, we could define clear levels of abstraction in a formula, which would be helpful to coordinate simultaneous expansion as discussed in the previous paragraph.

In a similar vein, we currently have no means of indicating how much content is collapsed in a particular position. A more precisely defined measure of complexity could help for this as well.

Size vs Content While collapsing content usually reduces the space used by an expression, this is not always the case. In particular, in deeply nested expressions, recursive collapses might make sense from a semantic point of view, but might not always be necessary for conserving space.

We observe this phenomenon with the expansion of an identity of Ramanujan given on the right hand side. We can see that already after the second or third step of the expansion there is hardly any space reduction in the size of the formula, while we are effectively abstracting homogeneously over the very similar formulas under the fraction. Thus, if we were purely interested in visual display, if sufficient space is available it would not make sense to go through all the single expansion steps.

$$\frac{1}{\blacktriangleleft\cdot\blacktriangleright} = 1 + \blacktriangleleft/\blacktriangleright$$

$$\frac{1}{\left(\blacktriangleleft\sqrt{}\blacktriangleright - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \blacktriangleleft/\blacktriangleright}$$

$$\frac{1}{\left(\blacktriangleleft\sqrt{}\blacktriangleright - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\blacktriangleleft/\blacktriangleright}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\blacktriangleleft/\blacktriangleright}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\blacktriangleleft/\blacktriangleright}}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\dots}}}}$$

On the other hand, to demonstrate the effect of the infinite recursion in the formula, e.g., for teaching or for the purpose of summarising the formula, the above sequence of collapse actions is perfectly suitably. Consequently, actions should be supported on different levels, those that primarily aim towards responsive visual rendering, versus those that allow for meaningful step-wise exploration of the content.

Accessibility One major goal of our work is to provide enhanced facilities to make Mathematics on the web fully accessible for people with visual impairments and other print disabilities. The collapse approach allows already reduction of complexity of a structure that can aid readers with print disabilities, like dyslexia. In addition, the summarisation effect should be exploitable to provide advanced explanations for structures via aural rendering. The current semantic approach has been originally developed in the context of the ChromeVox screen reader and forms the core of the Maths to speech translation in the MathML cloud project (cf. mathmlcloud.org). With the new embedded format it should now also be able to summarise expressions and translate sub-expressions on the fly, regardless of whether or not a screen reader can handle MathML.

Content MathML At the moment our semantic enrichment process provides enhanced presentation MathML, only. However, there is no theoretical hurdle in turning the current semantic structure into full blown Content MathML. While we give many symbols, like operators and relations, a default type and role, we stop short of mapping them to actual semantic meaning in the sense of determining, for example, for a plus symbol that it is addition between numbers or elements of an algebraic structure. One future step could be to attempt this step and generate content MathML, possibly taking additional context information into account. However, given the breadth of MathML content on the web and its (lack of) quality, we assume that generated Content MathML will be rather poor in many cases.

Electronic Media Appendix

We have provided a number of web sites with demonstrators for responsive equations.

mathjax.github.io/MathJax-RespEq/Semantics-Lab/Struik.html contains excerpts from lectures on Classical Differential Geometry. All display style equations are responsive either by mouse click or by reducing display size.
mathjax.github.io/MathJax-RespEq/Semantics-Lab/Semantics-Lab-MML-linebreaking.html and mathjax.github.io/MathJax-RespEq/Semantics-Lab/Semantics-Lab-TeX-linebreaking.html are pages to experiment with input expressions. Observe that the latter two sites are development test sites and therefore subject to code changes.

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